Distance Matching Notes

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July 29, 2021

1 Stop Prediction

To calculate the distance for the capsule to stop, we begin with Newton's second law:

$$\mathbf{F}_{\text{net}} = m\mathbf{a}, \tag{1}$$

where $\mathbf{F}_{\text{net}}$ is the net force[1] on the capsule, m is the mass of the capsule[2], and $\mathbf{a}$ is the acceleration. To make the presentation simpler, we will restrict our attention to a single dimension—the x dimension, say—such that

$$F_{\text{net},x} = m\dot{v}, \tag{2}$$

where $\dot{v}$ represents the acceleration in the x direction[3].

The two forces applied when the player input is zero are the force due to friction and the braking force, both of which resist the capsule's motion, meaning (2) becomes

$$-F_B - \gamma v = m\dot{v},$$

where F_B is the constant braking force and γ is the Stokes's drag coefficient, which has units of kilograms/second. But, if we look closely at the Character Movement Component (CMC), we notice that the units of the friction are 1/seconds. To rectify our equation to be more obviously analogous to Epic's, we can write

$$\begin{aligned} \dot{v} &= -\frac{\gamma}{m}v - \frac{F_B}{m} \\ &= -\beta v - a_B \end{aligned} \tag{3}$$

where $\beta = \gamma/m$ (and therefore has units of 1/seconds)and is the `Friction` we find in the CMC, and $a_B = F_B/m$ is the `BrakingAcceleration`—which is assumed, perhaps incorrectly, to be a constant[4]. The next step is to discretize equation (3) into partitions in time, each over a uniform time step size Δt, so we can put the equation into code[5].

There are many different ways to discretize a continuous equation, and all correct approaches will yield the same result in the limit of $\Delta t \to 0$. Below, I choose one approach.

[1] The net force is simply the sum of all the forces acting on the object of mass m.

[2] By default, the capsule mass is 100 kg in the Character Movement Component.

[3] Each dot above the variable represents a time derivative (following Newton's convention), so therefore both $\dot{v}$ and $\ddot{x}$ can represent the second derivative of x—hence, the acceleration along the x direction, or a_x.

[4] In this write-up, we take $a_B \geq 0$.

[5] We discretize time into equally-intervals of size Δt:

$$[0, t_1],\ [t_1, t_2], \ldots, [t_{i-1}, t_i], \ldots,$$

with $t_i = i\Delta t$.

We take equation (3), multiply both sides by the infinitesimal time element dt, and integrate[6] over a finite time interval of Δt:

$$\int_{v_{i-1}}^{v_i} dv = -\beta \int_{t_{i-1}}^{t_i} v\,dt - a_B \int_{t_{i-1}}^{t_i} dt.$$

Evaluating the right-hand side using the left-endpoint approximation gives

$$v_i - v_{i-1} = -\beta v_{i-1}\Delta t - a_B \Delta t.$$

Solving for velocity at the current time step leaves us with

$$\boxed{v_i = v_{i-1} - (a_B + \beta v_{i-1})\Delta t.} \tag{4}$$

Using a simple finite difference equation to relate position to velocity, where

$$v_i = \frac{x_i - x_{i-1}}{\Delta t},$$

we see that

$$\begin{aligned} x_i &= x_{i-1} + v_i\,\Delta t \\ &= x_{i-1} + \Big[v_{i-1} - (a_B + \beta v_{i-1})\Delta t\Big]\Delta t, \end{aligned}$$

and therefore the current position as a function of the previous position and the previous velocity is

$$\boxed{x_i = x_{i-1} + v_{i-1}\Delta t - (a_B + \beta v_{i-1})(\Delta t)^2.} \tag{5}$$

[6]The notation used is $v_i = v(t_i)$, and likewise for other non-time quantities.